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IEAP
Extraterrestrische Physik

Abstract:

We use a modified wavelet transformation to analyze ACE MAG data for the rate of occurrence of magnetic fluctuations. We analyze the magnetic field data for amplitude and width of individual waves and obtain distributions of occurring amplitudes and their dependence on the waves frequency and phase.
In the analyzed frequency range from 0.0001 to 0.5 Hz we find an exponential distribution of wave amplitudes.
The average wave amplitude increases with decreasing frequency and increasing solar activity. We find a significant fraction of large amplitude waves, which do not agree with the assumption of quasilinear theory that the irregularities of the magnetic field are sufficiently small and therefore the changes of an energetic particles pitch angle are also small.

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Wave amplitudes in the solar wind at 1 AU: Implications for energetic particle transport

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Method of data analysis:

The continuous wavelet transform is defined as [*Percival, 2000*]

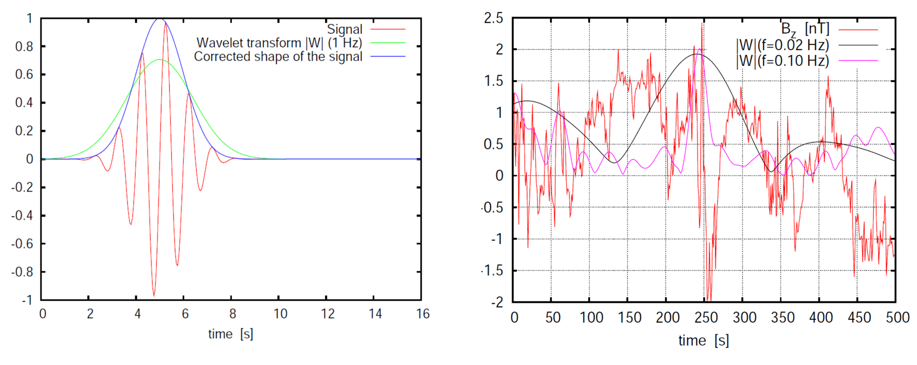
$$W(\lambda,\tau) := \int_{-\infty}^{\infty} x(t)\,\frac{1}{\sqrt{\lambda}}\psi\left(\frac{t-\tau}{\lambda}\right)$$

For our analysis we choose the Morlet wavelet

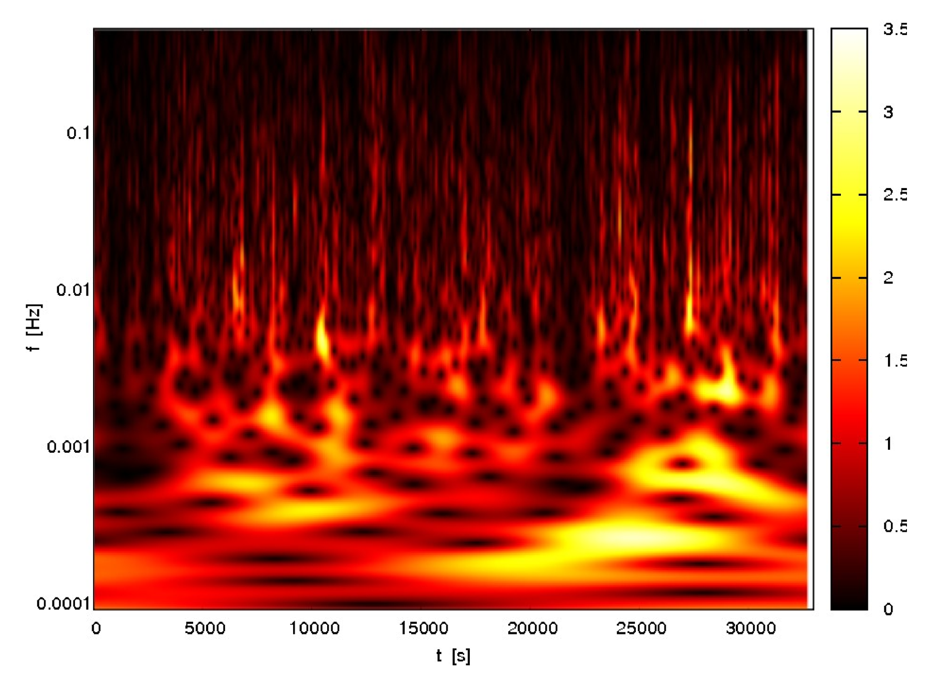
$$\psi(t) = \pi^{-1/4} e^{-i2\pi f_0 t} e^{-t^2/2}$$

To identify amplitudes and widths of short wave pulses in a signal, we require the wavelet analysis to return the correct amplitude of an oscillation.

$$W(\lambda,\tau) := \int_{-\infty}^{\infty} x(t)\,\sqrt{\frac{2}{\pi\lambda^2}}\psi\left(\frac{t-\tau}{\lambda}\right)$$
$$\psi(t) := e^{-i2\pi f_0 t} e^{-t^2/2}.$$



Wavelet analysis of a Gaussian pulse (left), analysis of ACE MAG data (right)



Intensity of frequencies over time for ACE MAG data

The wavelet analysis of a Gaussian shaped oscillation results in a Gaussian shape with a lesser amplitude and an increased standard deviation.
This can be understood if one considers that the wavelet transform is basically the convolution of signal and wavelet. Therefore the original shape can be reconstructed via

$$\sigma_B = \sqrt{\sigma_W^2 - \sigma_\psi^2}$$
$$B_{max} = W_{max} \cdot \frac{\sigma_W}{\sigma_\psi}$$

where σ_B,σ_W, σ_ψ are width of the wave, measured signal and wavelet. B_{max}, W_{max} are the max amplitude of signal and wavelet analysis.

We study 1s ACE MAG data, to obtain statistics $p_f(B_{max})$ of the occurring wave amplitudes and their dependence on frequency, solar wind type and the solar cycle.

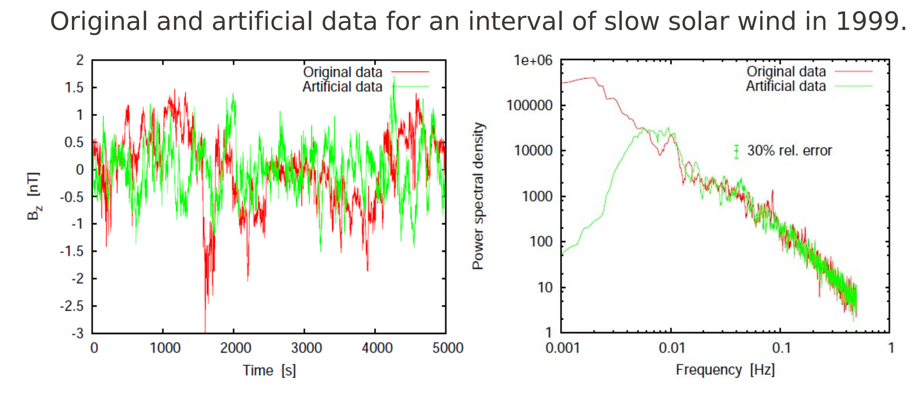
Wave distribution in the solar wind:

To demonstrate that the obtained wave amplitudes $b_{f,i}$ contain the information about the power-spectral density, we create artificial fluctuations from the amplitude distribution and compare original and artificial data.
The artificial signal will be created by

$$B(t) = \sum_f \sum_i b_{f,i} \sin(2\pi f \cdot t + \phi_{f,i}) \cdot e^{-\frac{(t-t_{f,i})}{2\sigma_{f,i}^2}}$$

where f are the analyzed frequencies, $b_{f,i}$ the detected wave amplitudes and $\Phi_{f,i}$, $t_{f,i}$ are random phase and positions of the artificial wave.

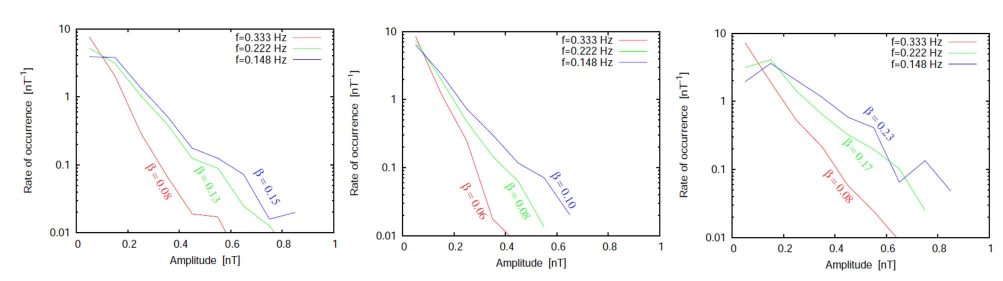
Original and artificial data for an interval of slow solar wind in 1999.



- Original and artificial data are similar
- Same power-spectral density for a wide range of frequencies
- At low frequencies the wavelet width has the same order of magnitude as the signal length, therefore the wavelet analysis can not be applied.

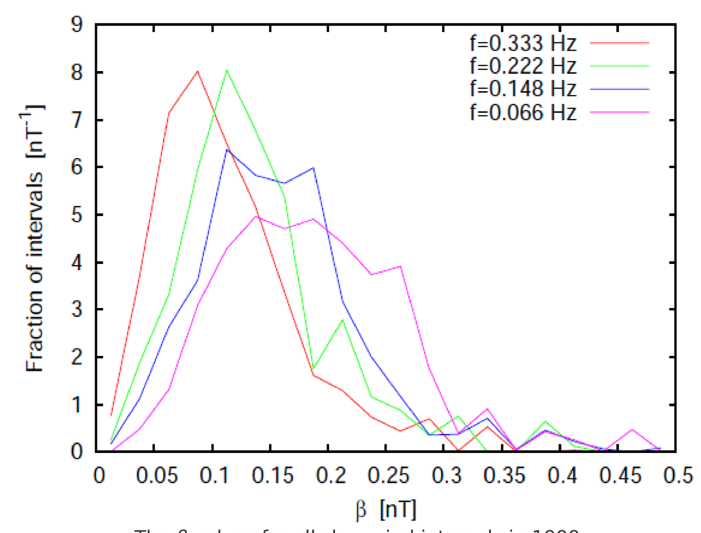
Amplitude distribution:

To get an impression of the amplitude distribution $p_f(B_{max})$ in the solar wind, we analyze the amplitudes perpendicular to the mean magnetic field in several solar wind intervals



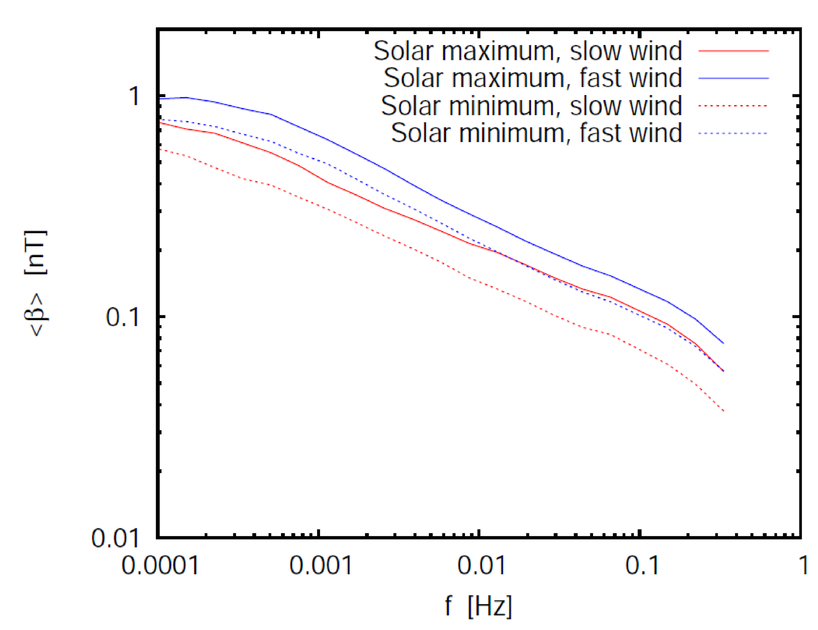
- Wave amplitudes are exponentially distributed
- The mean amplitude β increases with decreasing frequency
- The mean amplitude varies for different intervals

This exponential distribution occurs independent of solar cycle, solar wind type or frequency. The mean amplitude varies significantly for each solar wind interval



The β values for all slow wind intervals in 1999

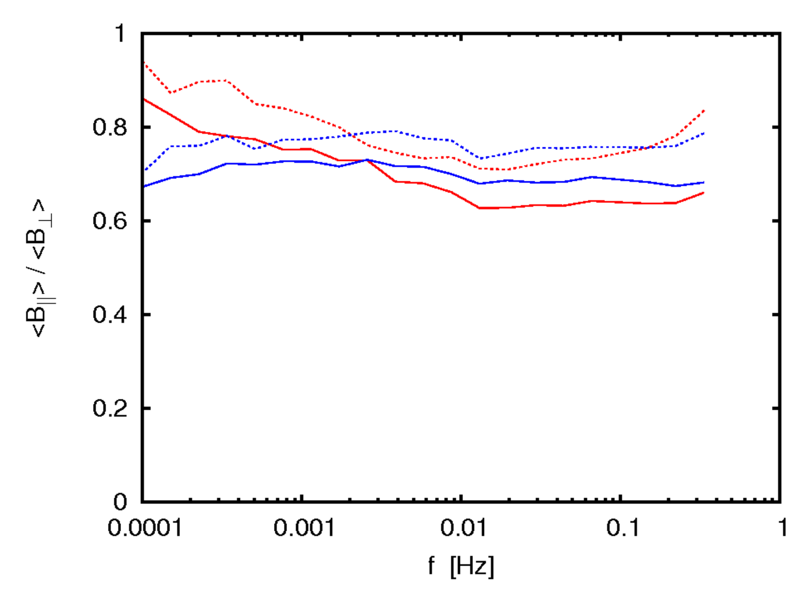
In a next step we systematically calculate the mean amplitude *perpendicular* to the mean magnetic field β for each slow and fast wind interval during solar maximum (1999-2003) and during solar minimum (2005-2008).
The average mean amplitude $<\beta>$ is calculated for frequencies in a range of 0.0001 – 0.5 Hz.



Mean amplitudes perpendicular to the average magnetic field

- Increase of wave amplitudes towards low frequencies
- Decreased amplitudes for slow wind and for solar minimum

Repeating this analysis for amplitudes *parallel* to the mean magnetic field yields a ratio of $<\beta_{||}> / <\beta_{\perp}> \sim 0.8$

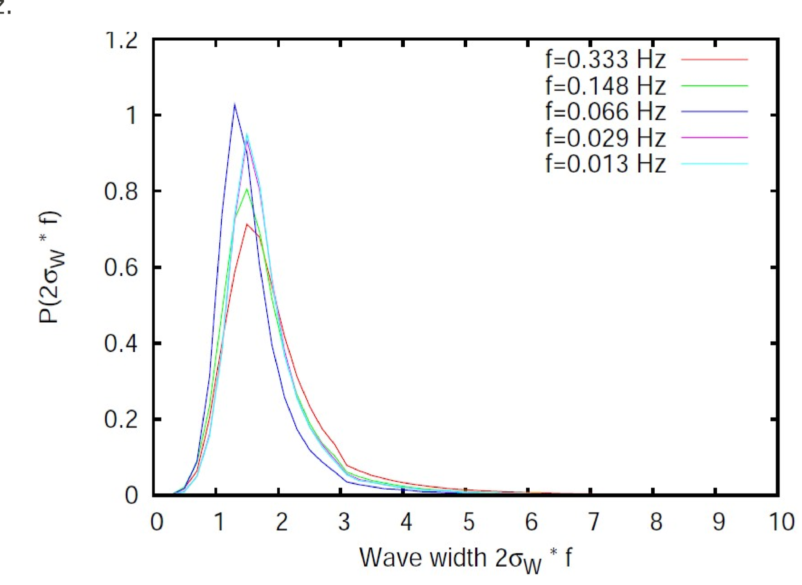


Amplitudes parallel to the mean magnetic field compared to amplitudes perpendicular to the mean magnetic field

Although not shown here, we can generate an interval of artificial fluctuations. The power spectral density of the artificial data exhibits the same spectral index of −5/3 as the measured data.

Wave width:

The width of the peaks in the wavelet analysis depends on the width of the used wavelet. The temporal resolution of the analysis increases with decreasing width of the wavelet.
We analyze the occurring wave width $2\sigma_W$ for a small wavelet with f_0 = 1 Hz.

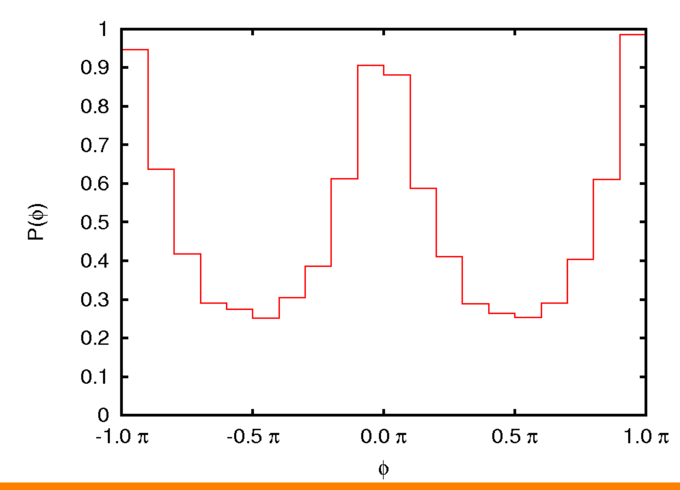


Polarization of waves:

Alfvén waves which propagate parallel to the mean magnetic field can be classified by their sense of rotation with respect to the mean magnetic field. L-/R-mode waves rotate in a left/right hand direction with respect to the mean magnetic field. Analyzing two perpendicular components of the magnetic fluctuations which are perpendicular to the nominal Parker field, one can use the wavelet transformation to calculate the mode for individual waves. The angle in the complex plane φ between the two wavelet transformations $W_{Bz}(\tau)$ and $W_{B\perp}(\tau)$ states the polarization of a wave at position τ .
For example, the angle between the wavelet transformations of

$$B_z = \cos(\omega t)$$
$$B_\perp = \cos(\omega t + \phi)$$

is given by φ, where $\varphi = 0, \pm\pi$ is a linear polarized wave and $\varphi = \pm\pi/2$ is a circular polarized wave.



Summary:

- The modified wavelet analysis can be used to analyze a signal for the occurrence of fluctuations with a certain frequency
- The statistics about *width* and *amplitude* of the individual oscillations contains the information to create a similar signal with the same power-spectral density
- Amplitudes in ACE MAG data show an *exponential distribution*
- The mean amplitude decreases towards high frequencies and is increased during fast solar wind and solar maximum
- The mean amplitudes parallel to the magnetic field are decreased by a factor of 0.8 compared to the mean amplitudes perpendicular to the mean magnetic field
- A distribution of wave widths can be obtained, but depends significantly on the shape of the wavelet
- A large fraction of the detected waves are linearly polarized or consist of 2D structures (~ 70%)
- Circular polarized waves (~ 30%) display amplitudes which are decreased by a factor of 0.8 compared to the linear polarized waves.

Conclusion:

- Depending on solar wind type the $< \beta >$ values range from 0.05 nT at 0.333 Hz up to 0.8 nT at 10^{-3} Hz
- As a particular example, for a frequency of f = 0.002 Hz during solar maximum with fast solar wind, $< \beta >$ equals 0.5 nT
- Considering the high variability of $< \beta >$, a fraction of ~ 15% of all intervals shows mean amplitudes > 1 nT.
- Assuming an exponential distribution, ~15% of the occuring 0.002 Hz waves have amplitudes > 2 nT

Energetic particles interact with those fluctuations with which they are in resonance [*Jokipii , 1966; Hasselmann & Wibberenz, 1968*]. One of the assumption underlying quasilinear theory is, that the *irregularities of the magnetic field are sufficiently small* that the changes of the energetic particle's pitch angle during a single gyration are small [*Dröge, 1994, 2003*].
As show in the above estimation, a significant fraction of solar wind intervals contains 0.002 Hz waves with amplitudes above 2 nT. For an ambient 6 nT magnetic field a resonant 2 nT oscillation will create *large changes* in an energetic particle's *pitch angle* which does not agree with the diffusive behavior of the quasilinear theory.

Thus, our investigations demonstrate the need to include the effects of wave particle scattering by large-amplitude waves in the transport equation for energetic particles. One possible approach is to include *jump processes* in addition to the usual diffusive pitch-angle scattering, as for example in the differential Chapman-Kolmogorov Equation [*Gardiner, 1983*].

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